Detection of periodicity in functional time series

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Periodicity is one of the most important characteristics of time series, and tests for periodicity go back to the very origins of the field. The importance of such tests has manifold reason. One of them is that most inferential procedures require that the series be stationary, but classical stationarity tests (as e.g. KPSS procedures) have little power against a periodic component in the mean.

In this account we respond to the need to develop periodicity tests for functional time series (FTS). Examples of FTS's include annual temperature or smoothed precipitation curves, daily pollution level curves, various daily curves derived from high frequency asset price data, daily bond yield curves, daily vehicle traffic curves and many others.

One of the important contributions of this article is the development of a *fully functional ANOVA test for stationary data*. If the functional time series (Y_t) satisfies a certain weak-dependence condition, then, using a frequency domain approach, we obtain the asymptotic null-distribution (for the constant mean hypothesis) of the functional ANOVA statistic. The limiting distribution has an interesting form and can be written as a sum of independent hypoexponential variables whose parameters are eigenvalues of the spectral density operator of (Y_t). To the best of our knowledge, there exists no comparable asymptotic result in FDA literature.

Adapting ANOVA for dependence is one way to conduct periodicity analysis. It is suitable when the periodic component has no particular form. If, however, the alternative is more specific or the period is large then we can construct simpler and more powerful tests. We hence introduce three different models with increasing complexity and develop the appropriate test statistics. The power-advantage will be illustrated in simulations and by a theoretical case study where we consider local consistency results for three specific alternatives.

A common approach to inference for functional data is to project observations onto a low dimensional basis system and then to apply a suitable multivariate procedure to the vector of projections. This approach will also be explained and discussed.

The talk is based on joint work with Piotr Kokoszka (Colorado State University) and Gilles Nisol (ULB).