Problems on discrete event simulation

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1. **Nonhomogeneous Poisson process** The number of calls to an insurance company announcing a car accident through a given day follows a nonhomogeneous Poisson process with intensity function $\lambda(x) = 1000x^4(1-x)$, where $0 \leq x \leq 1$ is given in days.
   a. What is the distribution of the total number of calls in a day (specify parameter).
   b. Simulate the phone calls received in 10000 days (10000 paths). For each day, determine the shortest time period between two consecutive phone calls. For any given day, what is the probability that two calls (the closest ones in time) are received within a time gap of 10 seconds.
2. **Compound Poisson process** The number of purchases at a webpage follows a nonhomogeneous Poisson process with intensity function $\lambda(x) = x$ during the first 10 days and $\lambda(x) = 10$ purchases per day ever after.
   a. Simulate 10000 processes until 100 purchases are executed. What is the average time until those 100 purchases?
   b. Three different products are sold, one for 200 euro, another for 300 euro, and the third one for 500 euro. Half of the purchases correspond to the first product, 30% to the second, and the remaining to the third. What is the expected time until they sell products for a total value of 25000 euro?
3. **Pricing European options** European options may only be exercised at expiration. That is, if we buy today a European call option with strike price $k = 100$ and maturity $t_m = 1$ year, in one year (at maturity) we have the right to buy the asset for the fixed strike price $k = 100$, which we will do if the asset price at that precise moment is greater than k. If the price of the asset at that moment is below k, we will not exercise the option. Suppose we want to price a European call option with the initial asset price $S(0) = 100$, strike price $k = 100$, risk-free rate $r = 0.02$, volatility $\sigma = 0.25$, and maturity $t_m = 1$ year. The risk-neutral pricing process is

$$S(t) = S(0)\exp\left((r - \sigma^2/2)t + \sigma Z\sqrt{t}\right),$$

   where $Z \sim \mathrm{N}(0,1)$. Use $MC = 10000$ simulations to price the option and give a 95% CI on the price. The option will only be exercised if the asset's price at maturity is greater than 100, so its payoff will be $\max\{S(t_m) - k, 0\}$, while the price is the expected payoff. Observe that the price is to be payed today, so it must be given in today's price of money, and we can use the risk-free rate to determine today's price of 100 monetary units in one year, which is $100\exp(-r) = 98.01987$.