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1. Random experiments

Problem 1. Given three events A, B, and C, express the following events using the union, intersection and complementary event

1. Out of the three events, only A occurs.
2. The three events occur.
3. A and B occur, but C does not.
4. At least two out of the three event occur.
5. Exactly two of the three events occur.
6. At most two of the three events occur.
7. At least one of the three events occurs.
8. One and only one of the three events occurs.
9. None of them occur.
10. If $P(A) = 0.2$, $P(B) = 0.4$, $P(C) = 0.3$, $P(A \cap B) = 0.1$ and $(A \cup B) \cap C = \emptyset$, which is the probability of each of the events described in the 9 previous expressions?

Problem 2. Complete the expression $P(A \cup B \cup C) = P(A) + P(B) + P(C) \cdots$

Problem 3. A *flight number*, when combined with the name of the airline and the date, identifies a particular flight. The Federal Aviation Administration limits flight numbers to four digits (0001 through 9999). Eastbound and northbound flights are assigned even numbers, while westbound and southbound flights have odd numbers. If all possible flight numbers for an eastbound or northbound flight are equally likely, what is the probability that the flight number of an eastbound or northbound flight generated at random is 1562?

Problem 4. At a randomized controlled trial (RCT) each study subject is randomly allocated to receive one or other of the alternative treatments under study. Assume five subjects undergo a RCT. Each of them tosses three fair coins, if he obtains exactly one head, he is given a placebo, otherwise, he is treated with the drug under study. What is the probability that four of the subjects are treated with the drug and one with the placebo?

Problem 5. Consider a random experiment consisting on tossing twice a biased coin that results in heads 60% of the tosses. Are the events $H_1 \equiv$"head at the first toss" and $E \equiv$"equal result at both tosses" independent? (explain why).

Use R package `prob` to simulate 1000 times the previous experiment (set your NIU, DNI, or NIE as seed and type your code and results). What fraction of experiments resulted in head at the first toss?, what fraction of experiments resulted in identical outcomes at both tosses? If we restrict to those experiments with identical outcomes at both tosses, what fraction of them resulted in head at the first toss? Does the empirical evidence about the independence of H_1 and E coincide with your answer to the first question?

Problem 6. An insurance company has clients classified as high, medium, and low risk. These clients have a probability of claiming equal to 0.02, 0.01, and 0.0025 respectively. If the probability of being a client of high risk is 0.1, 0.2 of medium risk, and 0.7 of low risk, what is the probability that a claim selected at random comes from a high risk client?

Problem 7. A company aims to predict the likelihood of a flight arrival delay up to six hours before airlines notify passengers by crunching data on weather, a flight's prior inbound airplane's status, FAA updates, historical data and other information. Assume that 20% of all flights suffer some kind of delay and that the company predicts as delayed 50% of the flights that are actually delayed, while it (wrongly) predicts as delayed 5% of the flights landing on time.

a) If you are taking three flights on three different days this summer, what is the probability that at least one of them is delayed? In case you needed to make any assumption, explain which.

b) What percentage of flights are announced to be delayed by the company?

c) If the company announces a delay at a given flight, what is the probability that it is actually delayed?