Optimal portfolios with minimum capital requirements

Abstract

We propose a novel approach to active risk management based on the recent Basel II regulations to obtain optimal portfolios with minimum capital requirements under alternative stress scenarios. In order to avoid regulatory penalties due to an excessive number of value-at-risk (VaR) violations, capital requirements are minimized subject to a given number of violations over the previous trading year. An empirical application for two portfolios involving different types of assets and alternative stress scenarios demonstrates that the proposed approach delivers an improved balance between capital requirement levels and the number of VaR exceedances. Furthermore, the risk-adjusted performance of the proposed approach is superior to that of minimum-VaR and minimum-stressed VaR portfolios.

Keywords: , Convex optimization, Multivariate GARCH, out-of-sample evaluation, stress testing.
JEL Classification: G11, G32

1. Introduction

The Basel II framework (Bank for International Settlements, 2006) requires banks to set aside a minimum amount of regulatory capital to cover potential losses arising from their exposure to market risk, credit risk, and operational risk. Market risk is the risk of losses on positions in equities, interest rate related instruments, currencies and commodities due to adverse movements in market prices. The capital requirement (CR) for market risk is based upon estimates of the Value-at-Risk (VaR), defined as the maximum loss on the bank's positions in these assets that could occur over a given holding period with a specified confidence level. Recent changes in the Basel II regulations established an additional CR based upon a stressed VaR (sVaR), which reflects the risk on the bank's current portfolio if the relevant market factors were experiencing a period of stress; see Bank for International Settlements (2009).

Basel II allows banks to use 'internal' models to measure their VaR and sVaR, as an alternative to the standardized approach described in the accord (Hendricks and Hirtle, 1997).

This standardized approach is known to render conservative VaR estimates, leading to excessively high CR. From the banks' perspective this is undesirable given that, among others, regulatory capital involves an opportunity cost as it cannot be used for other, profitable purposes. Hence, it is attractive for banks to attempt to lower their capital charges using their own risk management system. The empirical evidence presented by Pérignon et al. (2008) suggests that the use of internal models indeed is widespread among large financial institutions.

Although internal risk measurement systems are subject to supervisory approval based on qualitative and quantitative standards, banks enjoy a large degree of freedom in devising the precise nature of their models. This flexibility does not, however, imply that banks are tempted to pursue the lowest possible VaR estimates. This is due to the fact that the relation between the VaR estimates and CR is non-monotonic, as it takes into account not only the magnitude of the VaR but also the number of VaR violations (i.e. actual losses exceeding the VaR) in the recent past. Specifically, the regulatory capital required to be held on day $t+1$ is determined as the maximum of the current VaR estimate and the average VaR over the preceding 60 business days multiplied by a scaling factor, that is,

$$\text{CR}_{t+1} = \max\left\{\text{VaR\$}_t(h,\alpha), (3+k) \times \overline{\text{VaR\$}_{t,60}(h,\alpha)}\right\}, \tag{1}$$

where $\text{VaR\$}_t(h,\alpha)$ is the estimate at day t of the VaR for a holding period of h days at confidence level $\alpha \in (0,1)$ and $\overline{\text{VaR\$}_{t,60}(h,\alpha)} = \frac{1}{60}\sum_{j=0}^{59} \text{VaR\$}_{t-j}(h,\alpha)$. Note that these VaR estimates are expressed in dollar terms, representing the loss that might be incurred on the current portfolio; that is, $\text{VaR\$}_t(h,\alpha) = V_t(1 - e^{\text{VaR}_t(h,\alpha)})$ with V_t being the current portfolio value and $\text{VaR}_t(h,\alpha)$ the VaR in terms of returns. Usually it is the latter VaR that is first obtained from a model for the portfolio return distribution, and we follow this practice here. The Basel II accord requires the use of VaR estimates for a holding period h of 10 days at confidence level α of 1%. Moreover, the accord allows the 10-day VaR estimates to be computed from VaR estimates for shorter periods by using the square-root-of-time-rule, that is $\text{VaR}_t(10,\alpha) = \sqrt{10/h}\,\text{VaR}_t(h,\alpha)$ for some $h < 10$, see Bank for International Settlements

(2006, paragraph 718(Lxxvi)).[1] The penalty or "plus" k in the multiplication factor in (1) ranges between 0 and 1. Its exact value is determined by the number of VaR exceedances during the last 250 business days, as shown in Table 1.

[Insert Table 1 here.]

During the financial crisis of 2007/2008, losses in most banks' trading books have been substantially larger than the VaR-based minimum CR determined according to (1). In response, the Bank of International Settlements (BIS) released a set of modifications for the existing regulatory framework regarding market risk; see Bank for International Settlements (2009). Among the main adjustments are the introduction of the sVaR and a corresponding new CR formula that leads to higher CR levels. The new CR formula is

$$\begin{aligned} \mathrm{CR}_{t+1} = \max\left\{\mathrm{VaR\$}_t(h,\alpha), (3+k)\times\overline{\mathrm{VaR\$}_{t,60}(h,\alpha)}\right\} \\ + \max\left\{\mathrm{sVaR\$}_t(h,\alpha), (3+k)\times\overline{\mathrm{sVaR\$}_{t,60}(h,\alpha)}\right\}, \end{aligned} \quad (2)$$

where $\mathrm{sVaR\$}_t(h,\alpha)$ is the estimate at day t of the sVaR for a holding period of h days at confidence level $\alpha \in (0,1)$, $\overline{\mathrm{sVaR\$}_{t,60}(h,\alpha)} = \frac{1}{60}\sum_{j=0}^{59}\mathrm{sVaR\$}_{t-j}(h,\alpha)$. The new regulations state that the backtesting results applicable for calculating the penalty parameter k are based upon estimates of the VaR only and not on the sVaR. Finally, no particular methodology is prescribed for computing the sVaR, except that it should reflect the VaR of the bank's current portfolio under extreme adverse market conditions. We discuss in Sections 2.1 and 3.4 alternative approaches to obtain the sVaR.

The expressions for the CR in (1) and (2) seemingly suggest that lower capital charges could be achieved by lower VaR (and sVaR) estimates. This, however, need not be the case as lower VaR estimates are possibly violated more often, thus increasing the regulatory capital through the effects of the penalty factor k. Apart from direct costs due to the larger amount of capital that needs to be put aside, this may also bring indirect costs by damaging

[1] Diebold et al. (1998) and Danielsson and Zigrand (2006) discuss the use of the square root rule.

the bank's reputation. Both types of costs become particularly severe when the 'red zone' is entered, that is, when ten or more VaR violations occur during a period of 250 business days. In that case, the bank may be forced to adopt the Basel accord's standardized approach for VaR estimation. As noted before, this approach is known to render conservative VaR estimates, leading to excessively high CR. In addition, the ban of the bank's internal models obviously has detrimental effects on its reputation.

In practice banks appear to be wary of being overly optimistic about their level of market risk during tranquil periods. In fact, empirical evidence presented by Berkowitz and O'Brien (2002), Pérignon et al. (2008) and Pérignon and Smith (2010) suggests that they systematically *overestimate* their VaR. For instance, Berkowitz and O'Brien (2002) document that the number of violations of VaR estimates of six large US banks is usually lower than expected. Similarly, Pérignon et al. (2008) report that for VaR estimates at the 1% level of the six largest Canadian banks there are only two violations during the 7,354 trading days analyzed, whereas the expected number is 74. The opposite situation seems to occur in times of stressed market conditions. During the 2007/2008 financial crisis, banks systematically *underestimated* their VaR and their level of market risk; see Bank for International Settlements (2009). This alternation of over- and underestimation of market risk levels may, at least to some extent, be due to the fact that VaR measures typically are calibrated using historical data. Following a period of calm in financial markets, the VaR estimates and the accompanying CR can decline to low levels, but then might underestimate risk during a period of stress that lies ahead. In fact, one of the main motivations for the introduction of the sVaR in the amendments to the Basel II accord is to reduce the procyclicality of the minimum CR. In addition, European regulatory institutions performed a number stress tests to assess the resilience of the banking system to absorb shocks on credit, market and sovereign risks and introduced further modifications in the regulatory framework; see Committee of European Banking Supervisors (2010).

The exaggeration of banks' own level of risk during normal times implies an excessive amount of regulatory capital, directly affecting the profitability of the bank. Another, at least as undesirable consequence is that such banks appear more risky than they actually are, thus generating reputational concerns about their risk management systems. Similarly, the underestimation of banks' own level of risk during times of stressed market conditions may lead to insufficient amount of regulatory capital to cover potential losses in the trading book, thus increasing the risk of bankruptcy. This affects investors' perception and can induce underinvestment in VaR-overstating and VaR-understating banks. Indeed, Jorion (2002) shows that VaR disclosures are informative about the future variability in trading revenues, thus corroborating the idea that analysts/investors may be using the VaR measures to support investment decisions.

In this paper we put forward a novel portfolio construction methodology to overcome the drawbacks of both over- and understatement of a bank's VaR. Specifically, we propose to determine optimal portfolio weights by directly minimizing the daily CR, but subject to a restriction on the number of VaR violations during the preceding year. Implicitly, our approach aims to find the optimal balance between the level of VaR measures and the number of VaR violations, thus leading to the lowest possible level of CR.

Although minimizing CR is an important criterion to take into account, in real world situations portfolio managers and investors traditionally decide upon their asset allocations by considering standard performance measures, such as expected returns or Sharpe ratios. In addition, portfolio weights often are restricted in order to avoid shortselling or to limit the exposure to individual assets. For this reason we consider a general formulation of the portfolio construction problem in which the optimal portfolio composition is found by minimizing the level of CR subject also to a given (i.e. user specified) target performance and to direct constraints on the portfolio weights.

We apply the proposed methodology to two different asset portfolios: (i) a mixed portfolio composed of 30 futures on a variety of assets including equities, bonds, commodities and currencies, and (ii) an equity portfolio comprising 48 US industry indices. The minimum capital requirement (MCR) portfolio is compared to various benchmark portfolios, including the minimum-VaR portfolio (Alexander and Baptista, 2002), the minimum-sVaR portfolio, and the equally weighted (1/N) portfolio. In our empirical analysis we pay particular attention to the consequences of the introduction of the sVaR-based CR. For this purpose, in addition to 'normal' market conditions we consider several alternative, realistic scenarios in which expected returns, volatilities and cross-correlations are modified to reflect a stressed environment. Furthermore, we consider different models for obtaining forecasts of expected returns, volatilities and correlations, which are crucial inputs for the asset allocation decisions. We also examine the robustness of our results to the specific restrictions imposed on the portfolio's target rate of return and the re-balancing frequency.

The results for the futures portfolio indicate that our approach delivers lower CR levels in comparison to the benchmark portfolios. For the portfolios of sector indices, the novel portfolio construction approach delivers a better balance between CR levels and the number of VaR violations, as it yields a lower average number of VaR exceedances. For both data sets, we find that the number of VaR violations under the MCR portfolio policy does not enter the red zone in any of the (normal and) stress scenarios and for all of the specifications used for forecasting volatilities and correlations. This is in sharp contrast to the benchmark portfolios, for which we frequently find more than ten VaR violations. Finally, the performance of the MCR portfolios in terms of risk-adjusted returns and portfolio turnover is generally superior to the minimum-VaR and minimum-sVaR portfolios.

Our proposed methodology differs in important ways from previous, related research. First, in order to achieve the goal of lower CR, one possibility is to develop a VaR model that delivers lower levels of capital charges, as proposed recently by McAleer et al. (2010), for instance. Using the terminology of Christoffersen (2009), this approach can be considered a risk measurement or *passive* risk management approach, since it is applied to a given (i.e. predetermined) portfolio composition. Alternatively, in this paper we propose to perform *active* risk management by deciding on the portfolio allocations themselves to attain lower levels of CR.

Second, portfolios with low levels of CR may be obtained by imposing constraints on the amount of CR or on the portfolio VaR, as in Sentana (2003), Cuoco and Liu (2006) and

Alexander et al. (2007). In our approach, the level of CR plays a much more central role as it is taken to be the objective function that should be minimized.

The remainder of the paper is organized as follows. In Section 2 we describe the procedure to obtain the optimal portfolios with minimum CR subject to restrictions on the number of VaR violations. In Section 3 we present the empirical applications. We conclude in Section 4.

2. Portfolios with Minimum Capital Requirements

The main ingredient required to obtain optimal portfolios with minimum capital requirements (hereafter MCR portfolios) is a measure of the VaR and of the sVaR. Therefore, in Section 2.1 we first describe the procedure for obtaining VaR and sVaR estimates considered in this paper. In Section 2.2 we then develop the optimization problem that leads to the construction of MCR portfolios.

2.1. VaR and sVaR estimation

Denote by $R_{t+h} = (r_{1,t+h}, \ldots, r_{N,t+h})'$ the vector of h-period returns (between t and $t+h$) of the N assets that may be included in the portfolio. The portfolio return is given by $r_{p,t+h} = w_t' R_{t+h}$, where w_t is the vector of portfolio weights to be determined at time t. The portfolio VaR at time t for a given holding period h and confidence level α is given by the α-quantile of the conditional distribution of the portfolio return. Thus, $\text{VaR}_t(h, \alpha) = F^{-1}_{p,t+h}(\alpha/100)$, where $F^{-1}_{p,t+h}$ is the inverse of the cumulative distribution function of $r_{p,t+h}$. Throughout the paper we focus on the portfolio VaR for a holding period of $h = 1$ day at $\alpha = 1\%$. The latter is the relevant confidence level that banks must adopt in computing their risk exposure. As mentioned before, the Basel accord requires the use of VaR estimates for a holding period h of 10 days, but it allows these to be computed from VaR estimates for shorter periods by using the square-root-of-time-rule. Therefore, from now on we omit the arguments h and α from the definition of the VaR.

When the distribution of returns is expressed in terms of its two first conditional moments, the portfolio return can be represented as

$$r_{p,t+1} = \mu_{p,t+1} + \sigma_{p,t+1}\varepsilon_{p,t+1} \tag{3}$$

where $\mu_{p,t+1}$ and $\sigma_{p,t+1}$ are the conditional mean and standard deviation of the portfolio return, given by

$$\mu_{p,t+1} = w_t'\mu_{t+1} \tag{4}$$

and

$$\sigma_{p,t+1}^2 = w_t'H_{t+1}w_t, \tag{5}$$

where $\mu_{t+1} = \mathsf{E}[R_{t+1}|R_1,\ldots,R_t]$ is the $N \times 1$ vector of conditional mean returns for the N individual assets and H_{t+1} is the $N \times N$ conditional covariance matrix, $H_{t+1} = \mathsf{E}[(R_{t+1} - \mu_{t+1})'(R_{t+1} - \mu_{t+1})|R_1,\ldots,R_t]$. The standardized unexpected returns $\varepsilon_{p,t+1}$ in (3) are independent and identically distributed with mean equal to zero and unit variance, i.e. $\mathsf{E}[\varepsilon_{p,t+1}] = 0$ and $\mathsf{E}[\varepsilon_{p,t+1}^2] = 1$ for all t. The portfolio VaR is then given by

$$\text{VaR}_{t+1} = \mu_{p,t+1} + \sigma_{p,t+1}q \tag{6}$$

where q is the α-quantile of the distribution of $\varepsilon_{p,t+1}$.

The novel CR specification in (2) requires the estimation of the sVaR. As mentioned before, the sVaR is similar to the 'normal' VaR, except that it should measure the risk of extreme losses if the relevant market factors were experiencing a period of stress. Accordingly we define the sVaR as:

$$\text{sVaR}_t = \tilde{\mu}_{p,t+1} + \tilde{\sigma}_{p,t+1}q \tag{7}$$

where $\tilde{\mu}_{p,t+1} = w'\tilde{\mu}_{t+1}$, $\tilde{\sigma}_{p,t+1} = (w'\tilde{H}_{t+1}w)^{1/2}$, and $\tilde{\mu}$ and $\tilde{H}$ are the vector of stressed conditional expected returns and the stressed conditional covariance matrix, respectively. The amendments to the Basel accord do not provide specific implementation details concerning the stress scenarios, except that they should typically involve lower expected returns, higher volatilities and more extreme correlations. In Section 3.4 we discuss the specification of alternative stress scenarios used in the empirical analysis.

The VaR definition in (6) requires estimates of the conditional expected returns μ_{t+1}, the conditional covariance matrix H_{t+1} and the quantile q or, more generally, the distribution of the standardized unexpected returns. For each of these three inputs, both nonparametric and parametric specifications may be adopted. Given that in this paper we focus on high-dimensional portfolios consisting of a large number of assets N, parametric specifications may be more appropriate. In this case, the expected returns may be obtained from linear (vector) autoregressive [(V)AR] models as well as nonlinear models (Carriero et al., 2009; Pesaran et al., 2009; DeMiguel et al., 2010). Similarly, alternative specifications for the conditional covariance matrix H_{t+1} can be considered, including multivariate GARCH models (see Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) for comprehensive reviews), stochastic volatility models (Harvey et al., 1994; Aguilar and West, 2000; Chib et al., 2009), as well as realized covariance matrices based on high-frequency intraday data (De Pooter et al., 2008; Barndorff-Nielsen et al., 2008). Finally, the models for the conditional mean and variance are usually estimated by assuming a particular distribution for $\varepsilon_{p,t+1}$, such as the normal or the Student's t distribution. This distribution may also be considered in order to obtain the quantile q in (6). For instance, when assuming normality of $\varepsilon_{p,t+1}$, $q = -2.33$ for $\alpha = 1\%$. See Santos et al. (2009) for an empirical comparison among alternative procedures for computing the inverse of the cumulative distribution function of the portfolio returns.

Finally, it is worth noting that the MCR portfolio construction methodology developed in the remainder of this section is independent of the method used to obtain the VaR and sVaR measures. However, and perhaps obviously, we may expect that more accurate modeling of the expected returns and conditional covariance matrix leads to improved portfolio characteristics.

2.2. MCR Portfolios

The problem of constructing an MCR portfolio consists of finding the vector of portfolio weights w that minimizes the capital charges subject to a restriction on the number of VaR exceptions during the previous 250 trading days and other constraints. We now describe the objective function as well as the restrictions involved in the optimization problem in more detail.

Objective function

The objective function for constructing an MCR portfolio consists of minimizing the amount of regulatory capital given by the 'original' Basel II formulation in (1) or by the 'new' specification in (2). To save space we focus on the latter, noting that it is straightforward to formulate a objective function using the CR formula in (1). For the sake of convenience, we rephrase the function given in (2) in terms of the VaR and the sVaR expressed in portfolio returns. Using the VaR and sVaR definitions in (6) and (7) and the expressions for the conditional mean and variance of the portfolio returns in (4) and (5), we can write the objective function as

$$\begin{aligned}\underset{w}{\text{minimize}}\ &\max\left\{-(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q), -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q)\right\} \\ &+ \max\left\{-(w'\tilde{\mu}_{t+1} + (w'\tilde{H}_{t+1}w)^{1/2}q), -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\tilde{\mu}_{t+1-j} + (w'\tilde{H}_{t+1-j}w)^{1/2}q)\right\}. \quad (8)\end{aligned}$$

In the first (second) term of (8), we take the maximum of minus the current VaR (sVaR) estimate and the average over the previous 60 business days. Note that for computing the average VaR and sVaR over the previous 60 business days we need the historic one-step-ahead forecasts for i) the conditional mean and for the stressed conditional mean, μ_{t+1-j} and $\tilde{\mu}_{t+1-j}$, and ii) the conditional covariances and the stressed conditional covariances, H_{t+1-j} and $\tilde{H}_{t+1-j}$, for $j = 0, \ldots, 59$. Even more important to note is that the average VaR and sVaR are hypothetical, in the sense that they are based on the portfolio with the weights that are currently determined for day $t+1$.

Restriction on the number of VaR violations

A VaR violation occurs when the portfolio return on a given trading day falls below the VaR estimate. This can be characterized by means of an indicator function, $\mathbf{1}\big(w'R_{t+1} < w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q\big)$, which takes the value 1 when the argument is true, i.e. when a VaR violation occurs on day $t+1$. We restrict the number of VaR exceedances over the last 250 trading days to be less than or equal to a certain threshold δ, that is, we impose the restriction

$$\sum_{j=1}^{250} \mathbf{1}\big(w'R_{t+1-j} < w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q\big) \leq \delta. \tag{9}$$

The value of δ can be chosen by taking into consideration the penalties reported in Table 1. For instance, if we intend to avoid the number of VaR violations reaching the "red zone", we should set $\delta = 9$. Similar to the average VaR and sVaR over the previous 60 business days in (8), the number of VaR violations over the previous 250 days in (9) is hypothetical, in the sense that it is based on the portfolio with the weights that are currently determined for day $t+1$.[2] It is also important to note that we include a restriction on the number of VaR violations and not on the number of sVaR violations. This is in line with the current regulations, which establish that backtesting results applicable for calculating the penalty parameter k in (2) are based upon estimates of the VaR only and not on the sVaR; see Bank for International Settlements (2009).

Target performance

In many practical situations involving portfolio selection, investors and portfolio managers are interested in achieving a certain target performance. For that purpose, we incorporate the following restriction on the expected portfolio returns:

$$w'\mu_{t+1} \geq \Xi, \tag{10}$$

[2]Of course, the ex-post evaluation of the portfolio in terms of the number of VaR violations and capital requirements is based on actual portfolios, therefore in accordance to the criteria established by Basel II. Further implementation details are discussed in Section 3.5.

where Ξ denotes the desired target performance. Note that alternative specifications for restrictions on the target performance can be considered, such as a constraint on the Sharpe ratio, on the portfolio turnover, or on the tracking error; see Cornuejols and Tütüncü (2007).

Constraints on the portfolio weights

Finally, restrictions often are imposed on the portfolio weights to avoid short-selling or to achieve a minimum diversification level. Previous research has shown that imposing such constraints may substantially improve performance, mostly by reducing risk, see Jagannathan and Ma (2003), among others. For this reason, we allow for a general set of constraints on the portfolio weights, formulated as

$$w \in \Omega, \tag{11}$$

where Ω represents the set of allowable portfolio weights as defined by the specific restrictions that are imposed. For instance, if short-selling is to be avoided, we may specify the restriction $w \geq 0$. Similarly, a certain level of diversification may be guaranteed by imposing an upper bound on the individual portfolio weights, i.e. $w_i \leq u$ for some $0 < u < 1$.[3] Finally, we may achieve full investment by restricting the portfolio weights to sum up to 1, i.e. $\iota' w = 1$, where ι is a vector of ones.

The optimization problem as formulated in (8)-(11) is very general as it allows i) different econometric specifications for the expected returns and for the conditional covariance matrix, ii) different threshold levels for the maximum number of VaR violations, iii) different types of restrictions on the target performance, iv) alternative stress scenarios for the computation of the sVaR, and v) various different constraints on the portfolio weights. Albeit very general, the formulation in (8)-(11) has a major shortcoming: the objective functions in (8), and the restriction on the number of VaR violations in (9) are both discontinuous and non-convex due to the presence of the max-operator and the indicator function, respectively. Note that the

[3] DeMiguel et al. (2009a) recently proposed a unifying approach based on constraints of the portfolio norms that nests several commonly applied restrictions as special cases, including the no-shortselling and diversification constraints.

non-convexity imposes important difficulties in terms of computational effort and a potential problem of local minima; see Nocedal and Wright (1999) and Boyd and Vandenbergue (2004). For this reason we next formulate a convex and continuous approximation to the original problem for which a highly accurate solution can be obtained with low computational effort.

2.3. A convex and continuous reformulation

Reformulating the objective function

In order to obtain a continuous and smooth objective function, (8) can be reformulated by introducing artificial variables to get rid of the max-operator. Specifically, the objective function in (8) can be equivalently expressed as the following linear optimization problem:

$$\underset{w,v_1,v_2}{\text{minimize}}\, v_1 + v_2 \tag{12}$$

$$\text{subject to:}$$

$$v_1 \geq -(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q)$$

$$v_1 \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q)$$

$$v_2 \geq -(w'\tilde{\mu}_{t+1} + (w'\tilde{H}_{t+1}w)^{1/2}q)$$

$$v_2 \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\tilde{\mu}_{t+1-j} + (w'\tilde{H}_{t+1-j}w)^{1/2}q),$$

thus yielding a continuous and convex expression. Note that the attractiveness of the reformulation in (12) is that it replaces the minimization of a nonlinear, non-smooth objective function by the minimization of a smooth, linear objective function with convex constraints (Nocedal and Wright, 1999).

Reformulating the restriction on the number of VaR violations

Due to the presence of an indicator function, the original restriction on the number of VaR violations in (9) is non-differentiable, discontinuous and non-convex. We propose a *convex approximation* by eliminating the indicator function while keeping its argument, which leads to

$$\frac{1}{250}\sum_{j=1}^{250}\left(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q - w'R_{t+1-j}\right) < \tilde{\delta}, \tag{13}$$

where $\tilde{\delta}$ is a parameter that must be calibrated in order to achieve the desired results regarding the number of VaR violations over the last 250 observations. The procedure to calibrate this parameter is detailed in Subsection 3.5. Note that under this approximation, our displeasure regarding a VaR violation grows as the constraint becomes "more violated". In other words, this approximation implies that VaR violations of greater magnitude are more penalized than those of less magnitude, which makes sense from a practical point of view. Note that this feature is not captured by the indicator function, as in that case VaR exceedances of smaller magnitude have the same importance as those of greater magnitude. Moreover, the proposed approximation is the best convex approximation to (9); see Boyd and Vandenbergue (2004).

In sum, based on the reformulation of the objective function in (12) and of the constraint on the number of VaR violations in (13), the optimization problem based on (8)-(11) admits the following convex reformulation:

$$
\begin{aligned}
&\underset{w,v_1,v_2}{\text{minimize}}\, v_1 + v_2 \\
&\text{subject to:} \\
&\ v_1 \geq -(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q) \\
&\ v_1 \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q) \\
&\ v_2 \geq -(w'\tilde{\mu}_{t+1} + (w'\tilde{H}_{t+1}w)^{1/2}q) \\
&\ v_2 \geq -\frac{(3+k)}{60}\sum_{j=0}^{59}(w'\tilde{\mu}_{t+1-j} + (w'\tilde{H}_{t+1-j}w)^{1/2}q) \\
&\ \frac{1}{250}\sum_{j=1}^{250}\left(w'\mu_{t+1-j} + (w'H_{t+1-j}w)^{1/2}q - w'R_{t+1-j}\right) < \tilde{\delta} \\
&\ w'\mu_{t+1} \geq \Xi \\
&\ w \in \Omega.
\end{aligned}
\tag{14}
$$

Note that the optimization problem (14) is a *second-order cone formulation* (Nocedal and Wright, 1999; Boyd and Vandenbergue, 2004; Grant and Boyd, 2008). Therefore, the problem can be accurately solved in practice with low computational effort.

A final, technical comment about the optimization problem in (14) concerns the penalty factor k. Since the MCR portfolios are dynamic in the sense that the optimal portfolio weights have to be re-calculated on a daily basis, the penalty k also should be updated based on the actual number of VaR violations over the previous 250 trading days. This means that a minimum of 250 realizations of the MCR portfolio returns and corresponding VaRs are required to start evaluating and updating k. In order to be conservative, during the first 250 observations we set k=1 in the computation of the capital requirements, which is the highest value that the penalty factor can take. After the 250th trading day, we update k according to the values presented in Table 1. Moreover, to ensure a consistent portfolio evaluation we focus on the capital requirements and the number of VaR violations obtained after the 250th day.

3. Empirical Application

We evaluate the performance of the MCR strategy for two portfolios with different types of assets. In the optimization problems in (14), we adopt a daily target portfolio return of $\Xi = 4$ bp, corresponding to an annual target return of 10%. Furthermore, we focus on the case in which only long positions are allowed by imposing a no-shortselling restriction, i.e. $w \geq 0$.

3.1. Data sets

We consider portfolios constructed from two sets of different types of assets.[4] The first set consists of 30 futures contracts on equity indices (S&P500, NASDAQ, DJIA, Canada 60, FTSE, CAC, DAX, IBEX, MIB, Nikkei, Hang Seng, SGX, Bovespa, IPC), 10-year government bonds (US, UK, Germany, and Japan), currencies (euro, British pound, Japanese yen, Canadian dollar, Swiss franc, Australian dollar, Mexican peso and Brazilian real) and commodities (gold, silver, wheat, and crude). For each contract we measure returns in dollars, and implement appropriate adjustments for roll-overs from one futures contract to the next. The second set of assets comprises 48 industry portfolios of US stocks.[5] For the two sets of assets we obtain daily observations from March 1, 2000 until July 31, 2008. Returns are computed as the differences in log prices. The effective sample sizes are equal to 2194 and 2156 observations, respectively.

3.2. Expected returns and conditional covariances

Computing a VaR measure for a given portfolio requires estimates of the expected returns and the conditional covariance matrix of the included assets, see (6). In our empirical application these inputs are obtained from multivariate parametric models.

[4] We considered a third data set consisting of all stocks that belonged to the S&P100 index during the complete sample period. This yields a total of 81 stocks. The results are very similar to those obtained with the US industry portfolios and therefore are not reported to save space.

[5] This data set was obtained from the web page of Kenneth French (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`)

Expected returns are obtained from a VAR(1) model for the return vector R_{t+1},

$$R_{t+1} = C + \Phi R_t + \varepsilon_{t+1} \tag{15}$$

where C is an $N \times 1$ vector of constants, Φ is the $N \times N$ autoregressive matrix, and ε_{t+1} is a vector of shocks (or unexpected returns), which are assumed to be temporally uncorrelated and normally distributed with a positive definite conditional covariance matrix H_{t+1}.

We consider two of the most popular specifications for the conditional covariance matrix: the Risk Metrics model and the dynamic conditional correlation (DCC) model of Engle (2002). Moreover, we also consider the (unconditional) shrinkage estimator of the sample covariance matrix proposed by Ledoit and Wolf (2003), motivated by its ability of dealing with the estimation error in large covariance matrices.

Widely used by practitioners, the Risk Metrics (RM) approach consists of an exponentially-weighted moving average scheme to model conditional covariances. In this approach, the conditional covariance matrix is given by

$$H_{t+1} = (1 - \lambda) R_t R_t' + \lambda H_t, \tag{16}$$

with the recommended value for the model parameter for daily returns being $\lambda = 0.94$.

Time-varying conditional correlation models are currently one of the most promising alternatives to model and forecast conditional covariances, see Engle and Sheppard (2001) for comprehensive theoretical and empirical analysis. One of their greatest advantages is that they have a smaller number of parameters than traditional multivariate models such as VEC and BEKK models, and therefore can be applied to problems involving a large number of assets. DCC models are based on the decomposition of the conditional covariance matrix H_{t+1} into conditional standard deviations and correlations, that is

$$H_{t+1} = D_{t+1} P_{t+1} D_{t+1} \tag{17}$$

where $D_{t+1} = \text{diag}\left(\sqrt{h_{1,t+1}}, \ldots, \sqrt{h_{N,t+1}}\right)$ with diag$(\cdot)$ being the operator that transforms a

$N \times 1$ vector into a $N \times N$ diagonal matrix. The conditional variances $h_{j,t+1}$, $j = 1, \dots, N$, are assumed to follow a standard univariate GARCH(1,1) model. P_{t+1} is a symmetric positive definite conditional correlation matrix with elements $\rho_{ij,t+1}$, where $\rho_{ii,t+1} = 1$, $i, j = 1, \dots, N$. In the DCC model the conditional correlation $\rho_{ij,t+1}$ is given by

$$\rho_{ij,t+1} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1} q_{jj,t+1}}} \tag{18}$$

where $q_{ij,t+1}$, $i, j = 1, \dots, N$, are collected into the $N \times N$ matrix Q_{t+1}, which is assumed to follow GARCH-type dynamics,

$$Q_{t+1} = (1 - \alpha - \beta)\,\bar{Q} + \alpha z_t z_t' + \beta Q_t \tag{19}$$

where $z_t = (z_{1t}, \dots, z_{Nt})$ with elements $z_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ being the standardized unexpected returns, $\bar{Q}$ is the $N \times N$ unconditional covariance matrix of z_t and α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.[6]

Our third and final approach to model the covariance matrix is the shrinkage estimator of Ledoit and Wolf (2003) (LW). Shrinkage estimators are becoming very popular in the portfolio construction literature due to their ability to reduce the estimation error in large covariance matrices. For instance, Ledoit and Wolf (2003) and Ledoit and Wolf (2004) report improved results in terms of portfolio performance when the shrinkage estimator is

[6] The parameters of the DCC model are usually estimated using the two-step procedure proposed by Engle and Sheppard (2001) and Sheppard (2003). However, Engle et al. (2008) point out that when the dimension of the portfolio increases, the two-step estimator can be severely downward biased due to an undiagnosed incidental parameter problem. Therefore, in this paper we estimate the parameters of the DCC model by the composite likelihood (CL) estimator proposed by Engle et al. (2008). Essentially, this method is based on the decomposition of the original estimation problem into many small subproblems in such a way the composite likelihood is constructed by summing up the quasi-likelihood of subset of assets. By summing up over many subsets, the CL estimator does not require the inversion of large covariance matrices. In the case where all distinct pairs of the returns in the system are used, the CL involves $O(K^2)$ calculations, which is much smaller than the $O(K^3)$ implied in the maximization of the standard likelihood function, where K is the number of unique pairs of data. Consequently, the CL estimator is much faster. Furthermore, Engle et al. (2008) show that the CL estimator is not affected by the incidental parameter problem. Instead of all distinct pairs, in this work, we implement the CL estimator using all contiguous pairs of data, which implies $O(K)$ calculations; the extensive Monte Carlo results reported in Engle et al. (2008) show that the choice of contiguous pairs can be successfully applied in problems involving hundreds of assets.

used vis-à-vis traditional estimators such as the sample covariance matrix. In this paper we consider the shrinkage estimator proposed by Ledoit and Wolf (2003), which is defined as an optimally weighted average of the sample covariance matrix and the covariance matrix based on Sharpe (1963) single-index model. The intuition behind this shrinkage estimator is to come up with an optimal convex combination between an unbiased covariance matrix estimator that may be subject to substantial estimation error (i.e. the sample covariance matrix) and another estimator that possibly is biased but has considerably less estimation error (i.e. the covariance matrix from the single factor model). In this model the returns of asset i are described by:

$$r_{it} = a_i + b_i r_{mt} + \upsilon_{i,t}, \tag{20}$$

where r_{mt} is the market portfolio return.[7] The residuals υ_i are assumed to be uncorrelated with market returns and to exhibit no serial correlation. The covariance matrix F of the returns R_t implied by this model is:

$$F = \sigma_m^2 bb' + \Delta, \tag{21}$$

where σ_m^2 is the variance of the market returns, b is the vector of slopes or factor loadings, and Δ is a diagonal matrix containing variances of the residuals υ_t. The shrinkage estimator of Ledoit and Wolf (denoted by H_{LW}) then is defined as

$$H_{LW} = \psi F + (1 - \psi)S, \tag{22}$$

where ψ is the shrinkage intensity and S is the sample covariance matrix. A closed-form solution for the optimal shrinkage intensity (minimizing the distance between the true and estimated covariance matrices based on the Frobenius norm) is provided by Ledoit and Wolf (2003).[8]

[7]We follow Ledoit and Wolf (2003) and consider that the composition of the market portfolio is given by an equally weighted combination of the assets belonging to the portfolio under consideration.
[8]Code for computing the optimal shrinkage intensity is available at `http://www.iew.uzh.ch/institute/`

3.3. Benchmark portfolios

We consider three alternative benchmarks for the purpose of comparison with the proposed MCR portfolios. The first benchmark is a minimum-VaR (Min-VaR) portfolio. This is obtained from an optimization problem in which the investor wishes to perform active portfolio management by minimizing the portfolio VaR subject to a target return and possible other restrictions.[9] The Min-VaR optimization problem is given by

$$\begin{aligned} &\underset{w}{\text{minimize}} \; -\left(w'\mu_{t+1} + (w'H_{t+1}w)^{1/2}q\right) \\ &\text{subject to:} \\ &\; w'\mu_{t+1} \geq \Xi \\ &\; w \in \Omega. \end{aligned} \tag{23}$$

In (23), we adopt the same target return and the no-shortselling restriction used to obtain MCR portfolios.

The second benchmark portfolio is a minimum-sVaR (Min-sVaR) portfolio. This is obtained from an optimization problem analogous to (23), but replacing the conditional mean and the conditional covariance in the objective function by their stressed counterparts $\tilde{\mu}_{t+1}$ and $\tilde{H}_{t+1}$. In the resulting Min-sVaR optimization problem we adopt the same target return and the no-shortselling constraint used to obtain MCR portfolios.

As a third benchmark we consider the equally weighted (or $1/N$) portfolio, which has been extensively studied in the empirical literature. For instance, DeMiguel et al. (2009b) find that the $1/N$ portfolio outperforms (in terms or Sharpe ratio and turnover) 14 widely used portfolio strategies, such as mean-variance and minimum variance. Therefore, it seems natural to compare our results against this simple but powerful portfolio in which all the

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`people/wolf/publications.html`.

[9]The properties of the Min-VaR portfolio have been extensively studied by Alexander and Baptista (2002). It is shown that the solution to the VaR minimization problem is always distinct from the solution to the variance minimization problem. Moreover, the Min-VaR portfolio at the 99% confidence level is a mean-variance efficient portfolio with expected returns greater than the expected return of the min-variance portfolio.

assets have the same weight. We note that the $1/N$ portfolio returns are independent of the method used to model and forecast the expected returns and the conditional covariance matrix. On the other hand, the VaR estimate, the level of capital requirements, and the number of VaR violations are affected by these methods.

3.4. Stress scenarios

The sVaR measures the risk of extreme losses if the relevant market factors were experiencing a period of stress. As mentioned before the amendments to the Basel accord do not provide specific implementation details concerning the stress scenarios, except that they should typically involve lower expected returns, higher volatilities and more extreme correlations in agreement with empirical evidence, see Alexander (2009, Chapter IV.7). We consider three alternative stress scenarios that reflect the impact of relevant changes or 'haircuts' to the conditional moments of the asset returns on the estimation of the sVaR.[10]

Stress scenario 1 (Expected returns)

In this scenario we apply a haircut to the conditional expected return, i.e. $\tilde{\mu}_t = \mu_t - \Delta_\mu$, where $\Delta_\mu > 0$ is the haircut level for the expected return. In this case, we will have $\text{sVaR}_t = w'\tilde{\mu}_t + (w'D_tR_tD_tw)^{1/2}q$.

Stress scenario 2 (Expected returns and volatilities)

In this case we lower the conditional expected returns as in scenario 1 but also apply a haircut to the volatilities, i.e. $\tilde{\mu}_t = \mu_t - \Delta_\mu$ and $\tilde{D}_t = D_t + \Delta_D$, where $\Delta_D > 0$ is the haircut level for the volatilities. In this case, we will have $\text{sVaR}_t = w'\tilde{\mu}_t + (w'\tilde{D}_tR_t\tilde{D}_tw)^{1/2}q$.

Stress scenario 3 (Expected returns, volatilities and correlations)

In this case we lower the conditional expected returns and increase the conditional volatilities as in scenario 2, but in addition we apply a haircut to the conditional correlations,

[10] In unreported results, we considered a fourth stress scenario in which only correlations are stressed. The results are similar to those obtained under the stress scenarios reported here and are available upon request.

i.e. $\tilde{R}_t = R_t + \Delta_R$, where Δ_R is the haircut level for the correlations. In this case, $\text{sVaR}_t = w'\tilde{\mu}_t + (w'\tilde{D}_t\tilde{R}_t\tilde{D}_t w)^{1/2}q$. An important issue that arises when "stressing" the correlation matrix is that the resulting matrix $\tilde{R}_t$ may not be positive definite. To circumvent this problem, we employ the approach proposed by Qi and Sun (2010), which is designed to obtain the nearest positive definite correlation matrix in the context of the sVaR.

Definition of the haircut parameters

In order to obtain the stressed conditional moments for each of the stress scenarios defined above, it is necessary to define specific values for the haircut levels to be applied to expected returns, volatilities, and correlations. Our choices for the haircut levels are similar to those considered in the stress testing exercise conducted by the European regulatory authorities, see Committee of European Banking Supervisors (2010). Specifically, we set the haircut applied to the expected returns such that they become equal to -20% (in annualized percentage points). Moreover, we set the haircut applied to the volatilities such that volatility for each individual asset doubles. Finally, we set the haircut applied to the correlations such that the correlations (which typically are positive) among each pair of assets doubles. Considering that some assets may already have correlations larger than 0.5, we choose to 'cap' the correlations at 0.95, i.e. set $\tilde{\rho}_{ijt} = \min(2 \cdot \rho_{ijt}, 0.95)$.

It is worth noting that, in the case of the futures portfolio, some currencies and commodities often are considered to be 'safe havens', so that during crisis periods their prices increase rather than decline. The same may also apply to (some) government bonds. If we make this assumption, we may increase the expected returns of those assets, rather than lower them. We assume that gold, silver, Swiss Franc, and US government bonds are such 'safe havens' and that their expected returns are increased by 20% (in annualized percentage points). Accordingly, we assume that the correlations with the equity index futures (and other bonds, currencies and commodities) fall by half. For the 'non-safe haven' bonds, currencies, and commodities, we may assume a decline in expected returns and an increase in correlation with the equity index futures using the same haircut levels discussed above.

3.5. Implementation details

We use a rolling window of $\tau = 1000$ observations to estimate the parameters of the models that are used for generating the expected returns and the conditional covariance matrix. The following stepwise procedure then is used to obtain optimal MCR portfolios:

1. Using the observations for $t = 1, \ldots, \tau$, estimate the coefficients in the VAR(1) model (15) and in the model for the conditional covariance matrix.
2. Compute the expected return $\mu_{\tau+1}$ and the conditional covariance matrix $H_{\tau+1}$, and their corresponding stressed counterparts, $\tilde{\mu}_{\tau+1}$ and $\tilde{H}_{\tau+1}$, respectively, according to each of the stressed scenarios discussed in Subsection 3.4.
3. Use the last 250 observations up to observation τ to solve the optimization problem in (14) and obtain the optimal MCR portfolio weights w_τ for day $\tau + 1$.
4. Compute the portfolio return for day $\tau + 1$ as $r_{p,\tau+1} = w_\tau' R_{\tau+1}$, the portfolio VaR as $\mathrm{VaR}_{\tau+1} = w_\tau' \mu_{\tau+1} + (w_\tau' H_{\tau+1} w_\tau)^{1/2} q$, and the portfolio sVaR as $\mathrm{sVaR}_{\tau+1} = w_\tau' \tilde{\mu}_{\tau+1} + (w_\tau' \tilde{H}_{\tau+1} w_\tau)^{1/2} q$.
5. Move to the next window with observations $t = 2, \ldots, \tau + 1$ and repeat steps 1 to 4 until the end of the sample is reached.

After completing these steps, we have a total of $T - \tau$ out-of-sample observations for the portfolio return and one-step-ahead estimates of the portfolio VaR, where T denotes the sample size.

It is useful to note the following two points, related to our choice of models for the conditional covariance matrix. First, the Risk Metrics approach does not involve any unknown coefficients as we set $\lambda = 0.94$. Second, the shrinkage estimator of Ledoit and Wolf (2003) assumes that the covariance matrix is constant during the estimation window. When the shrinkage estimator is computed using an estimation window up to observation τ, we set the conditional covariance matrix $H_{\tau+1}$ equal to the resulting estimate.

The performance of the MCR portfolios depends on a good choice of the parameter $\tilde{\delta}$ in (13), which controls the desired maximum number of VaR violations. In order to calibrate this parameter, we use the following cross validation procedure.[11] Using the parameter estimates in the models for the conditional mean and for the conditional covariance matrix obtained with the first window of $\tau = 1000$ observations, we solve the MCR porfolio optimization problem in (14) for $t = 250, \ldots, \tau - 1$ for a range of values of $\tilde{\delta}$. We then pick the value of $\tilde{\delta}$ that minimizes the average capital requirements such that for each observation the maximum number of VaR violations (over the most recent 250 days) is less than or equal to 9, which is the upper bound for the "yellow zone" according to Table 1. The selected value for $\tilde{\delta}$ is used to obtain the MCR portfolios for the remaining observations of the data set. This procedure is implemented separately for each of the models for the conditional covariance matrix discussed in section 3.2 and also for each of the stress-scenarios discussed in section 3.4.

3.6. Out-of-sample evaluation

Most important for the evaluation of the MCR portfolios are the characteristics of the daily capital requirement (DCR) and the number of VaR violations, both in an absolute sense and compared to the benchmark portfolios. For each specification of the covariance matrix (RM, DCC, and LW) we consider the mean daily capital requirement ("mean DCR"), the average number of VaR violations ("Mean Hit"), the maximum number of VaR violations ("Max Hit"), and the fraction of days for which the number of VaR violations is either in the "green zone" (i.e. below 5) or in the "red zone" (i.e. above 9). The statistics concerning the VaR exceedances are based on rolling periods of 250 out-of-sample observations, which is the time period established by the Basel II accord to evaluate the financial institutions' VaR disclosures.

We also examine the portfolios' performance in terms of the average gross return ($\hat{\mu}$), standard deviation of returns ($\hat{\sigma}$), Sharpe ratio (SR) and turnover. These statistics are

[11] See Efron and Gong (1983) for a detailed explanation and DeMiguel et al. (2009a) for an application in the context of portfolio optimization.

computed as

$$\hat{\mu} = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1} w_t' R_{t+1}$$
$$\hat{\sigma} = \sqrt{\frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}(w_t' R_{t+1} - \hat{\mu})^2}$$
$$SR = \frac{\hat{\mu}}{\hat{\sigma}}$$
$$\text{Turnover} = \frac{1}{T-\tau-1}\sum_{t=\tau}^{T-1}\sum_{j=1}^{N}(|w_{j,t+1} - w_{j,t^+}|),$$

where w_{j,t^+} is the portfolio weight in asset j at time $t+1$ but *before* rebalancing and $w_{j,t+1}$ is the desired portfolio weight in asset j at time $t+1$. As pointed out by DeMiguel et al. (2009b), turnover as defined above can be interpreted as the average fraction of wealth traded in each period.[12]

To measure the impact of transaction costs on the performance of the different portfolios, we consider the average portfolio returns net of transaction costs, $\hat{\mu}_{TC}$, defined as

$$\hat{\mu}_{TC} = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1}\left[(1 + w_t' R_{t+1})\left(1 - c\sum_{j=1}^{N}|w_{j,t+1} - w_{j,t^+}|\right) - 1\right] \tag{24}$$

where c is the fee to be paid for each transaction. Instead of assuming an arbitrary value of c, we report the value of the *breakeven* transaction cost (Han, 2006). In other words, we report the value of c that makes the average portfolio return net of transaction costs equal to zero. Note that, when comparing two alternative portfolio strategies, the one with a higher breakeven cost is to be preferred.

To test the hypothesis that the capital requirement levels, the number of VaR exceedances, and the Sharpe ratios obtained with the MCR portfolios and with the benchmark portfolios are equal, we follow DeMiguel et al. (2009a) and use the stationary bootstrap of

[12]Note that, in the case of an equally weighted (or $1/N$) portfolio composition, we have $w_{j,t} = w_{j,t+1} = 1/N$, but w_{j,t^+} may be different due to changes in asset prices between t and $t+1$.

Politis and Romano (1994) with B=1,000 bootstrap resamples and expected block length b=5.[13] The resulting bootstrap p-values are obtained using the methodology suggested in Ledoit and Wolf (2008, Remark 3.2).

3.7. Results

Baseline empirical analysis

We first consider a baseline empirical analysis based on a version of the MCR portfolios in (14) using as objective function the capital requirement formula in (1). Table 2 reports the daily capital requirements, the number of VaR violations, and the performance of each portfolio strategy in terms of average gross returns, standard deviation of returns, Sharpe ratio, turnover, and breakeven transaction costs. Returns, standard deviation and Sharpe ratios are annualized. Breakeven transaction costs are reported in basis points (bp) and returns are reported in percentages. Capital requirements were computed according to the original Basel II capital requirement formula in (1), which does not require the estimation of the sVaR.

[Insert Table 2 here.]

The results for the global futures portfolio in Panel A of Table 2 indicate that, when the Risk Metrics model was used, the MCR strategy delivers the lowest average capital requirement (1.10%) in comparison to all benchmark strategies. In terms of the average number of VaR violations, the best performance is achieved when the LW model was used, although all portfolio policies achieve both average and maximum number of exceedances below the upper bound of the 'yellow' zone. It is also worth noting that, except for the $1/N$ portfolio, the average gross returns fall short of the annualized target return of 10% in all specifications. The risk adjusted performance measured by the Sharpe ratio were statistically equivalent across all specifications. Turnover is substantially lower for the MCR

[13] We performed extensive robustness checks regarding the choice of the block length, using a range of values for b between 5 and 250. Regardless of the block length, the test results for the differences in capital requirements, VaR exceedances and Sharpe ratios are similar to those reported here.

portfolios compared to the Min-VaR portfolios, such that the breakeven transaction costs are considerably higher for the MCR portfolios. As expected, by far the lowest turnover is achieved by the $1/N$ portfolio since changes in portfolios compositions are solely due to changes in asset prices. In fact, for this data set the best performance in terms of returns, turnover and breakeven transaction costs is achieved by the 1/N portfolio. This result corroborates previous findings in the literature, such as DeMiguel et al. (2009b) regarding the outperformance of the 1/N portfolio vis-à-vis more sophisticated portfolio strategies.

The results for the US industry portfolios in panel B of Table 2 nicely illustrate the trade-off between capital requirement levels and the number of VaR violations. At first glance, one could argue that the MCR portfolios do not perform well, in the sense that they render higher DCR than both benchmark portfolios. This conclusion is, however, misleading: The lower capital charges for the benchmark portfolios come at the expense of a very high number of VaR violations, leading to a rather substantial fraction of days spent in the "red zone". As discussed before, this is highly undesirable due to potential damaging effects on the banks' reputation regarding their risk management systems. For instance, using the DCC model for the Min-VaR portfolio delivers an average and maximum number of VaR violations equal to 10.53 and 18, respectively, leading to a total of 34% of days in the "red zone". On the other hand, for the MCR strategy the corresponding numbers are 2.67 and 5, altogether avoiding the "red zone" during the entire out-of-sample period. In terms of portfolio performance, the results are largely in favor of the MCR portfolio strategy. First, the MCR portfolios deliver gross returns higher than the Min-VaR and $1/N$ portfolios in all specifications, and also higher than the annualized target return of 10% in two out of three specifications. Second, the risk-adjusted performance of the MCR portfolios is significantly higher than that of the Min-VaR portfolio when the DCC and LW models are used. Third, the turnover for the MCR portfolio is lower than for the Min-VaR portfolio in all situations. Fourth, higher gross returns and the lower turnover lead to breakeven transaction costs for the MCR portfolio that are much higher than those for the Min-VaR portfolio. Therefore, we conclude that for this data set the performance of the proposed portfolio policy is highly superior in comparison to the benchmark portfolios.

Empirical analysis based on stress scenarios

We report in Tables 3 to 5 the results for capital requirements and portfolio portfolio performance for all portfolio policies obtained under stress scenarios 1 to 3, respectively. The characteristics of the stress scenarios are discussed in Subsection 3.4. Capital requirements were computed according to the current Basel II capital requirement in (2), which does require the estimation of the sVaR. For each stress scenario, we obtain the sVaR estimates using the haircut values discussed in Subsection 3.4. The MCR portfolios were obtained by solving the optimization problem in (14). Finally, we do not consider the LW model for the covariances in the scenario-based analysis. The reason is that it is not clear how to perform a decomposition of the LW covariance matrix into standard deviations and correlations, as in the case of the DCC model.

In line with the baseline analysis discussed above, the results for the global futures portfolio reported in Panel A of Tables 3 to 5 indicate that when the Risk Metrics model was used, the MCR strategy delivers the lowest average capital requirement in comparison to all benchmark strategies and across all stress scenarios. We also observe that all portfolio strategies deliver similar Sharpe ratios in all stress-scenarios. However, the MCR portfolios systematically achieve lower turnover, such that the breakeven transaction costs are considerably higher for the MCR portfolios in all specifications and across all stress scenarios.

[Insert Table 3 here.]

[Insert Table 4 here.]

[Insert Table 5 here.]

The results for the US industry portfolios reported in Panel B of Tables 3 to 5 are even more favorable to the MCR strategy. The MCR portfolios achieve a better balance in terms of capital requirement levels and the number of VaR violations. In all stress scenarios, although the average DCR for the MCR strtategy is higher in comparison to the benchmark policies, the number of VaR exceedances under the MCR strategy is the lowest and never exceeds "yellow zone" upper bound. For instance, when the DCC model for the conditional covariance matrices, the average number of VaR violations under stress scenarios 1 to 3 are 2.67, 3.09, and 2.78. The same numbers for the Min-sVaR policy are 10.53, 9.66, and 8.50 which end up leading to a high fraction of days in the "red zone". Furthermore, the performance of the MCR strategy in terms of average gross returns and Sharpe ratio is consistently better in comparison to the competing strategies. Finally, we observe that in all stress scenarios the MCR strategy delivers lower turnover and considerably higher transaction costs in comparison to the Min-VaR and Min-sVaR policy.

To further illustrate the results, in Figures 1 to 3 we plot the evolution of the number of VaR violations and the daily capital charge for the MCR (dashed line), Min-VaR (solid blue line), and Min-sVaR (solid cyan line) portfolios when the DCC specification is used for the conditional covariance matrix across stress scenarios 1 to 3, respectively. We also plot a horizontal line indicating the threshold value for the "red zone" (9 VaR exceedances in the previous 250 trading days). These graphs show that for the global futures portfolio the DCR and the number of VaR violations for the three portfolio policies are rather similar. For the other US industry portfolios, however, we observe that while the DCR is higher for the MCR portfolios, the number of VaR violations are much lower and - equally important - remain below the "red zone" threshold during the complete out-of-sample period. The Min-VaR and Min-sVaR portfolios, in contrast, deliver a number of VaR violations that are deep in the "red zone" during a high fraction of the out-of-sample period.

[Insert Figure 1 here.]

[Insert Figure 2 here.]

[Insert Figure 3 here.]

It is worth pointing out two interesting results that emerge from the analysis based on stress scenarios. First, we observe that as we move from scenarios 1 to 3 the average DCR tend to increase for all portfolio policies. This result is expected since the level of 'stress' increases as we move from scenarios 1 to 3. Note that in scenario 1 we only stress expected returns, while in scenario 3 we stress expected returns, volatilities and correlations. Consequently, the DCR levels tend to increase. For instance, when the DCC model was used, the average DCR for the MCR strategy under stress scenarios 1 to 3 are 2.45%, 2.90%, and 3.02%, respectively. Finally, it is important to note that the performance of the Min-VaR and the $1/N$ strategies in terms of gross returns, Sharpe ratio and transaction costs are the same across all stress scenarios. This is due to the fact that the optimal portfolio weights resulting from these strategies are not determined using stressed conditional moments. On the other hand, the optimal portfolio weights under the MCR and Min-sVaR strategies do consider stress conditional moments in their formulation. Therefore, their performance vary across stress scenarios.

Another interesting result arising from our results is concerned with the performance of the $1/N$ portfolio. Previous studies found that this portfolio strategy outperforms several sophisticated portfolio strategies in terms of risk adjusted performance and transaction costs. While we corroborate this finding, from the risk management point of view, however, the performance of the $1/N$ portfolio may be not so promising. For instance, in the Global portfolio the equally weighted portfolio delivers much higher average and maximum number of VaR violations in comparison to the MCR and Min-VaR portfolios, leading to a higher fraction of days within the "red zone".

Summarizing the results in Tables 2 and 5, the optimal MCR portfolios outperform the benchmark portfolios in several aspects. First, the MCR portfolios achieve a better balance between capital requirement levels and the number of VaR violations in comparison to the benchmark portfolios. The average number of VaR violations under the MCR portfolio strategy is the lowest in the vast majority of the specifications for the three data sets. Second, and in contrast to the competing portfolio strategies, the maximum number of VaR violations for the MCR portfolio almost never exceeds the "yellow zone" upper bound. Third, MCR portfolios achieve a better performance in terms of gross returns and Sharpe ratios in comparison to the Min-VaR and Min-sVaR benchmarks in one of the data sets considered in the paper. Finally, turnover for the MCR portfolio is lower than for the Min-VaR and Min-sVaR portfolios in all specifications, which together with the higher gross returns results in substantially higher breakeven transaction costs.

3.8. Robustness checks

A potential criticism to our results presented above is that they could be driven by a specific choice of the portfolio target return, of the re-balancing frequency, or of the econometric specifications for the expected returns and the conditional covariance matrix. In order to rule out this possibility, in this section we perform an extensive sensitivity analysis to check the robustness of the MCR portfolio's performance to changes in each of those settings. We report the results of the robustness checks in the web-appendix that can be accessed in the link `https://sites.google.com/site/andreportela/MDCR_portfolios_JBF_web_appendix.pdf`. Each Table in the web-appendix reports the performance indicators (mean DCR, mean Hit, max Hit, fraction of days in "red zone" and in "green zone", average gross return, standard deviation, turnover, Sharpe ratio, and breakeven transaction costs), averaged across alternative specifications for the covariance matrix. The results of the robustness checks are reassuring. First, the sensitivity analysis considering daily target returns of 2, 4 and 6 bp (equivalent to annual target returns of 5, 10 and 15%) in Tables 1 and 2 of the web-appendix indicate that, for the futures portfolio, the performance of the MCR strategy in terms of average DCR and the number of VaR violations is similar to those obtained with the benchmark policies. However, the MCR strategy delivers lower portfolio turnover, such that the breakeven transaction costs are higher in comparison to the benchmark strategies. For the US industry portfolios, the MCR strategy delivers a lower average number of VaR violations, higher average gross returns, higher Sharpe ratio, lower portfolio turnover, and higher breakeven transaction costs in comparison to the Min-VaR and Min-sVaR policies. Finally, we conclude that an increase in the target return leads only to a less-than-proportional (i.e. marginal) increase in the average gross returns and in the average Sharpe ratios. Second, we consider a sensitivity analysis on the portfolio re-balancing frequency. The results discussed previously are based on the assumption that the investor adjusts her portfolio on a daily basis. The transaction costs incurred with such frequent trading can possibly deteriorate the net portfolio performance. Obviously this effect can be avoided by adjusting the portfolio less frequently, such as on a weekly or monthly basis, which in fact is done in practice by many institutional investors. A drawback of re-balancing the portfolio less frequently is that portfolio weights become outdated, which may harm the performance. The performance of the MCR, Min-VaR and Min-sVaR portfolio strategies under daily, weekly and monthly re-balancing frequencies are summarized in Tables 3 and 4 of the web-appendix. As expected, we find that lowering the re-balancing frequency results in a substantial reduction in portfolio turnover. Finally, we perform a robustness check on the model used for the expected returns. The VAR(1) model in (15) contains $N + N^2$ unknown coefficients. Its use for generating expected returns thus entails a large amount of estimation uncertainty for the values of N considered here. As a more parsimonious alternative we consider using univariate AR(1) models for each individual asset, i.e. the matrix Φ in (15) is restricted to be diagonal. A drawback of this simplification is that it ignores possible important cross-correlations among the assets in the portfolio. The results in Tables 5 and 6 in the web-appendix indicate that regardless of the specification used for the conditional covariance matrix, the MCR portfolios continue to perform better than the benchmark portfolios when expected returns are obtained from univariate AR(1) models. Compared to the results based on the unrestricted VAR(1) model, we observe that for the Global portfolio the average DCR and the average number of VaR violations tends to be higher when univariate AR(1) models are used. However, the results for the US industry portfolios are mixed. Finally, we find that in the vast majority of the cases, modeling expected returns with a AR(1) results in higher SR and higher break even transaction costs compared to those originally obtained with a VAR(1) model.

4. Concluding Remarks

Previous empirical studies have found that banks and other large financial institutions tend to overestimate the VaR of their asset portfolios during tranquil times and to underestimate their risk in times of stressed market conditions. The VaR overestimation results in prohibitive amounts of regulatory capital requirements, thus generating opportunity costs and giving rise to reputational concerns. On the other hand, it also is not attractive for banks to underestimate their risk levels as this may lead to an excessive number of VaR violations and higher-than-expected losses. In addition, the regulations in the Basel II Accord and the recent changes therein impose a penalty on the regulatory capital in case VaR exceedances occur too frequently, such that lower VaR estimates may actually increase capital requirements. In this paper we proposed a novel approach based on active portfolio selection that alleviates these problems. The methodology involves setting portfolio weights in order to minimize the level of capital requirements, subject to a restriction on the number of VaR exceedances and other constraints (involving the target performance of the portfolio, for example). An empirical application to two portfolios composed of different types of assets demonstrated that the developed approach is able to provide a much better balance between capital requirement levels and the number of VaR violations compared to the minimum-VaR portfolio, minimum-stressed VaR, and the 1/N portfolio under alternative, realistic stress scenarios. This result is robust to the specifications of the expected returns and of the conditional covariance matrix, to the level of target returns, and to the portfolio re-balancing frequency.

Acknowledgements

The authors thank the anonymous referee for the comments, which have led to substantial improvements in the paper. We are also grateful to Kees Bouwman, Newton da Costa Jr., Victor DeMiguel, Andreas Heinen, Erik Kole, Roberto Meurer, Alfonso Novales, Cristiano de Oliveira, Pilar Poncela, Antonio Sanvicente, Timo Teräsvirta, Helena Veiga, Pedro Valls, and seminar participants at Universidad Carlos III de Madrid, Econometric Institute - Erasmus University Rotterdam, Insper, EESP - Fundação Getúlio Vargas, Itaú-Unibanco, XI Meeting of the Brazilian Finance Society for helpful comments and suggestions.

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Tables and Figures

Table 1: Basel II penalty zones

Zone	Number of VaR violations	k
Green	0-4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	$>$10	1.00

Note: The number of VaR violations is based on the preceding 250 business days.

Table 2: Baseline empirical analysis: Daily capital requirements, number of VaR violations and portfolio performance. The Table reports the average daily capital requirement (Mean DCR), the average and maximum number of VaR violations (Mean Hit and Max Hit, respectively), and the fraction of days the number of VaR violations are within the "green zone" (i.e. below 5) and within the "red zone" (i.e. above 9). Capital requirements are measured in percentages and are based on the original capital requirement formula established in the Basel II Accord. The Table also reports the average gross portfolio return, the standard deviation of portfolio returns, portfolio turnover, the Sharpe ratio, and the break even transaction cost. Returns, standard deviation and Sharpe ratio are annualized. The annualized target return is 10%. Returns are reported in percentages and break even costs are reported in basis points. One, two, and three asterisks indicate that the statistic is significantly lower than that of the Min-VaR portfolio at the 10%, 5%, and 1% level, respectively. All figures are based on subsequent (rolling) periods of 250 out-of-sample observations.

	Capital Requirements					**Portfolio Performance**				
	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross returns (%)	Std. deviation	Sharpe ratio	Turnover	Break even cost (b.p.)
Panel A: Global futures portfolio										
Covariance model: DCC										
MCR	1.22	3.72	8	0.00	72.78	4.55	3.49	1.30	0.46	3.87
Min-VaR	1.23	2.96	6	0.00	76.27	4.40	3.80	1.16	0.69	2.50
1/N	2.84	4.45	8	0.00	56.36	11.62	8.76	1.33	0.01	715.00
Covariance model: Risk Metrics										
MCR	1.10***	3.65***	6	0.00	91.00	4.31	3.46	1.25	0.49	3.49
Min-VaR	1.20	6.16	8	0.00	34.96	5.17	3.95	1.31	0.79	2.56
1/N	3.10	4.45	8	0.00	64.19	11.62	8.76	1.33	0.01	715.00
Covariance model: Ledoit-Wolf										
MCR	1.41	1.18	6	0.00	96.08	3.47	3.82	0.91	0.44	3.08
Min-VaR	1.49	1.52	6	0.00	96.08	0.34	4.99	0.07	0.68	0.20
1/N	3.07	7.00	16	19.49	46.61	11.62	8.76	1.33	0.01	715.00
Panel B: US industry portfolios										
Covariance model: DCC										
MCR	6.81	2.67***	5	0.00	98.39	23.87	20.20	1.18*	0.34	25.58
Min-VaR	3.52	10.53	18	34.49	32.18	5.12	10.80	0.47	0.99	2.03
1/N	5.00	8.34	15	22.61	37.37	6.73	14.30	0.47	0.01	503.80
Covariance model: Risk Metrics										
MCR	6.63	4.44***	7	0.00	65.40	9.84	19.05	0.52	0.54	7.03
Min-VaR	3.41	11.09	16	48.67	28.84	7.11	10.50	0.68	1.04	2.66
1/N	5.09	6.12	10	5.31	46.14	6.73	14.30	0.47	0.01	503.80
Covariance model: Ledoit-Wolf										
MCR	6.81	3.50***	10	6.57	78.43	26.91	18.80	1.43***	0.37	26.33
Min-VaR	3.60	10.66	25	28.84	42.21	3.68	11.31	0.33	0.80	1.82
1/N	4.58	8.44	24	22.61	63.32	6.73	14.30	0.47	0.01	503.80

Table 3: Daily capital requirements, number of VaR violations and portfolio performance for stress scenario 1. The Table reports the average daily capital requirement (Mean DCR), the average and maximum number of VaR violations (Mean Hit and Max Hit, respectively), and the fraction of days the number of VaR violations are within the "green zone" (i.e. below 5) and within the "red zone" (i.e. above 9). Capital requirements are measured in percentages and are based on the current capital requirement formula established in the Basel II Accord. The Table also reports the average gross portfolio return, the standard deviation of portfolio returns, portfolio turnover, the Sharpe ratio, and the break even transaction cost. Returns, standard deviation and Sharpe ratio are annualized. The annualized target return is 10%. Returns are reported in percentages and break even costs are reported in basis points. One, two, and three asterisks indicate that the statistic is significantly lower than that of the Min-VaR portfolio at the 10%, 5%, and 1% level, respectively. All figures are based on subsequent (rolling) periods of 250 out-of-sample observations.

	Capital Requirements					**Portfolio Performance**				
	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross returns (%)	Std. deviation	Sharpe ratio	Turnover	Break even cost (b.p.)
Panel A: Global futures portfolio										
Covariance model: DCC										
MCR	2.45	3.72	8	0.00	72.78	4.46	3.50	1.28	0.45	3.87
Min-VaR	2.43	2.42	5	0.00	92.16	4.40	3.80	1.16	0.69	2.50
Min-sVaR	2.46	3.32	7	0.00	75.95	3.62	3.88	0.93	0.66	2.18
1/N	5.68	4.45	8	0.00	56.36	11.62	8.76	1.33	0.01	715.00
Covariance model: Risk Metrics										
MCR	2.19***	4.45***	10	1.27	66.74	4.67	3.75	1.25	0.43	4.22
Min-VaR	2.39	5.67	9	0.00	37.71	5.17	3.95	1.31	0.79	2.56
Min-sVaR	2.75	3.57***	5	0.00	88.03	2.91	7.29	0.40	0.99	1.17
1/N	4.08	4.45***	8	0.00	64.19	11.62	8.76	1.33	0.01	715.00
Panel B: US industry portfolios										
Covariance model: DCC										
MCR	13.12	2.67***	5	0.00	98.39	25.95	19.39	1.34**	0.33	28.63
Min-VaR	6.93	7.28	14	26.87	58.48	5.12	10.80	0.47	0.99	2.03
Min-sVaR	7.27	10.53	18	34.49	32.18	4.36	11.13	0.39	1.01	1.69
1/N	10.00	8.34	15	22.61	37.37	6.73	14.30	0.47	0.01	503.80
Covariance model: Risk Metrics										
MCR	10.35	2.94***	6	0.00	97.12	13.84	19.88	0.70	0.63	8.28
Min-VaR	6.03	9.29	14	31.49	28.84	7.11	10.50	0.68	1.04	2.66
Min-sVaR	5.23	7.03***	11	9.11	35.18	8.38	13.53	0.62	0.94	3.45
1/N	5.78***	6.12***	10	5.31	46.14	6.73	14.30	0.47	0.01	503.80

Table 4: Daily capital requirements, number of VaR violations and portfolio performance for stress scenario 2.

The Table reports the average daily capital requirement (Mean DCR), the average and maximum number of VaR violations (Mean Hit and Max Hit, respectively), and the fraction of days the number of VaR violations are within the "green zone" (i.e. below 5) and within the "red zone" (i.e. above 9). Capital requirements are measured in percentages and are based on the current capital requirement formula established in the Basel II Accord. The Table also reports the average gross portfolio return, the standard deviation of portfolio returns, portfolio turnover, the Sharpe ratio, and the break even transaction cost. Returns, standard deviation and Sharpe ratio are annualized. The annualized target return is 10%. Returns are reported in percentages and break even costs are reported in basis points. One, two, and three asterisks indicate that the statistic is significantly lower than that of the Min-VaR portfolio at the 10%, 5%, and 1% level, respectively. All figures are based on subsequent (rolling) periods of 250 out-of-sample observations.

	Capital Requirements					**Portfolio Performance**				
	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross returns (%)	Std. deviation	Sharpe ratio	Turnover	Break even cost (b.p.)
Panel A: Global futures portfolio										
Covariance model: DCC										
MCR	2.90***	3.09	5	0.00	91.84	4.10	3.84	1.07	0.46	3.54
Min-VaR	3.28	2.42	5	0.00	92.16	4.40	3.80	1.16	0.69	2.50
Min-sVaR	2.85***	2.37*	6	0.00	91.31	4.10	4.37	0.94	0.56	2.89
1/N	8.19	4.45	8	0.00	56.36	11.62	8.76	1.33	0.01	715.00
Covariance model: Risk Metrics										
MCR	2.57***	4.39***	7	0.00	59.43	4.40	3.92	1.12	0.43	4.01
Min-VaR	3.24	5.67	9	0.00	37.71	5.17	3.95	1.31	0.79	2.56
Min-sVaR	2.62*	3.30***	7	0.00	82.63	4.22	6.24	0.68	0.75	2.21
1/N	4.94	4.45***	8	0.00	64.19	11.62	8.76	1.33	0.01	715.00
Panel B: US industry portfolios										
Covariance model: DCC										
MCR	20.49	3.32***	6	0.00	83.28	18.81	19.39	0.97	0.37	19.07
Min-VaR	10.40	7.28	14	26.87	58.48	5.12	10.80	0.47	0.99	2.03
Min-sVaR	10.46	9.66	17	28.03	32.18	4.50	10.63	0.42	0.72	2.44
1/N	15.00	8.34	15	22.61	37.37	6.73	14.30	0.47	0.01	503.80
Covariance model: Risk Metrics										
MCR	13.97	2.30***	6	0.00	97.12	13.60	20.05	0.68	0.63	8.16
Min-VaR	8.50	9.29	14	31.49	28.84	7.11	10.50	0.68	1.04	2.66
Min-sVaR	5.64***	7.03***	11	9.11	35.18	8.05	13.39	0.60	0.73	4.27
1/N	6.46***	6.12***	10	5.31	46.14	6.73	14.30	0.47	0.01	503.80

Table 5: Daily capital requirements, number of VaR violations and portfolio performance for stress scenario 3.
The Table reports the average daily capital requirement (Mean DCR), the average and maximum number of VaR violations (Mean Hit and Max Hit, respectively), and the fraction of days the number of VaR violations are within the "green zone" (i.e. below 5) and within the "red zone" (i.e. above 9). Capital requirements are measured in percentages and are based on the current capital requirement formula established in the Basel II Accord. The Table also reports the average gross portfolio return, the standard deviation of portfolio returns, portfolio turnover, the Sharpe ratio, and the break even transaction cost. Returns, standard deviation and Sharpe ratio are annualized. The annualized target return is 10%. Returns are reported in percentages and break even costs are reported in basis points. One, two, and three asterisks indicate that the statistic is significantly lower than that of the Min-VaR portfolio at the 10%, 5%, and 1% level, respectively. All figures are based on subsequent (rolling) periods of 250 out-of-sample observations.

	Capital Requirements					**Portfolio Performance**				
	Mean DCR (%)	Mean Hit	Max Hit	% of days in red zone	% of days in green zone	Gross returns (%)	Std. deviation	Sharpe ratio	Turnover	Break even cost (b.p.)
Panel A: Global futures portfolio										
Covariance model: DCC										
MCR	3.02***	2.78	5	0.00	91.95	3.99	3.98	1.00	0.49	3.19
Min-VaR	3.55	2.42	5	0.00	92.16	4.40	3.80	1.16	0.69	2.50
Min-sVaR	2.92***	2.18***	6	0.00	98.52	3.86	4.67	0.83	0.58	2.61
1/N	9.67	4.45	8	0.00	56.36	11.62	8.76	1.33	0.01	715.00
Covariance model: Risk Metrics										
MCR	2.50***	4.00***	7	0.00	70.44	4.46	4.01	1.11	0.45	3.91
Min-VaR	3.21	5.67	9	0.00	37.71	5.17	3.95	1.31	0.79	2.56
Min-sVaR	3.41	3.65***	6	0.00	75.64	7.73	8.95	0.86	1.18	2.54
1/N	4.77	4.45***	8	0.00	64.19	11.62	8.76	1.33	0.01	715.00
Panel B: US industry portfolios										
Covariance model: DCC										
MCR	21.31	2.18***	5	0.00	98.15	18.78	20.62	0.91	0.38	18.50
Min-VaR	11.82	7.28	14	26.87	58.48	5.12	10.80	0.47	0.99	2.03
Min-sVaR	11.13**	8.50	17	28.37	41.64	0.46	10.62	0.04	0.71	0.26
1/N	17.60	8.34	15	22.61	37.37	6.73	14.30	0.47	0.01	503.80
Covariance model: Risk Metrics										
MCR	13.63	2.89***	5	0.00	95.16	15.14	20.02	0.76	0.64	8.89
Min-VaR	8.44	9.29	14	31.49	28.84	7.11	10.50	0.68	1.04	2.66
Min-sVaR	6.02***	7.92*	14	25.72	39.91	9.17	14.42	0.64	1.41	2.52
1/N	6.37***	6.12***	10	5.31	46.14	6.73	14.30	0.47	0.01	503.80

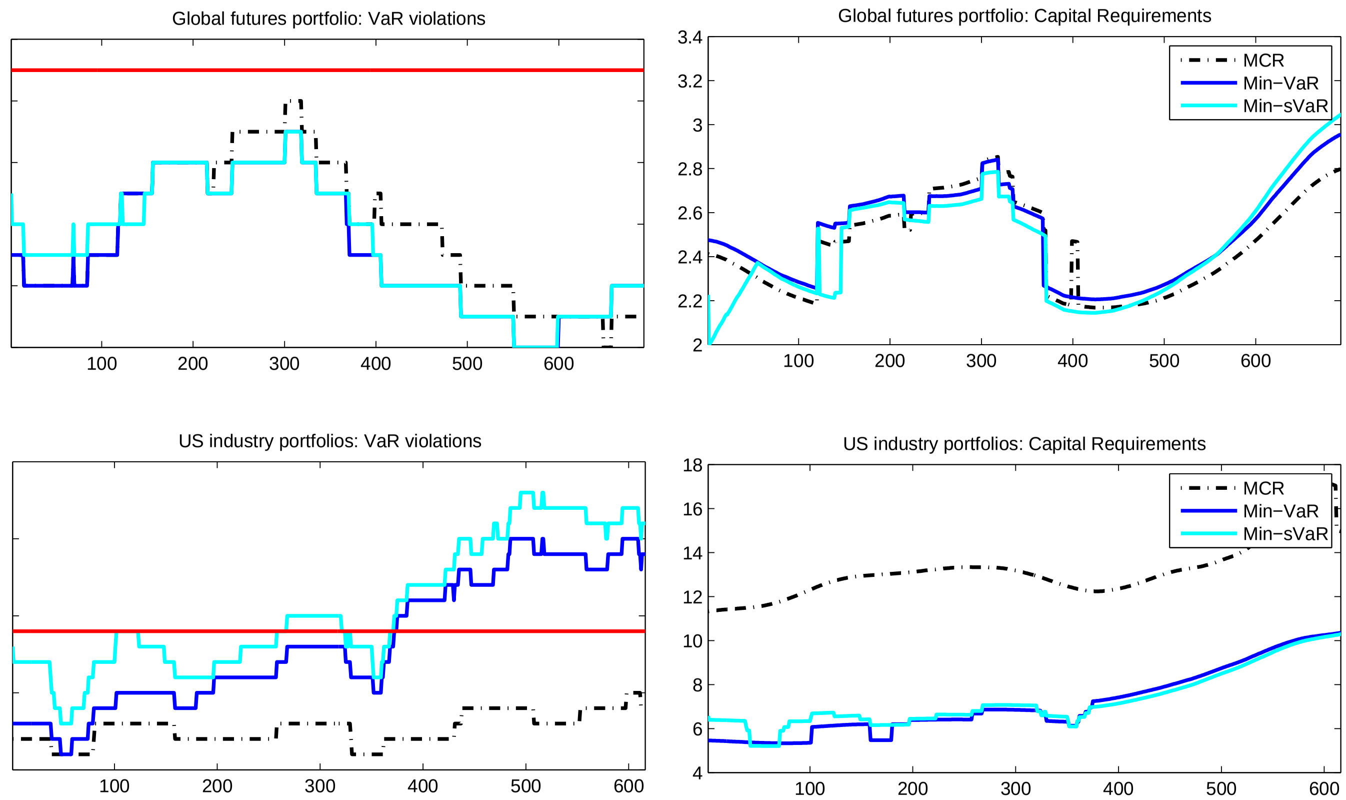

Figure 1: Number of VaR violations (left) and capital requirements (right) for the MCR (dashed line), Min-VaR (solid blue line), and Min-sVaR (solid cyan line) portfolios under stress scenario 1 when the DCC specification is used to model the conditional covariance matrix. The horizontal line indicates the “red zone” threshold (9 violations in the previous 250 trading days).

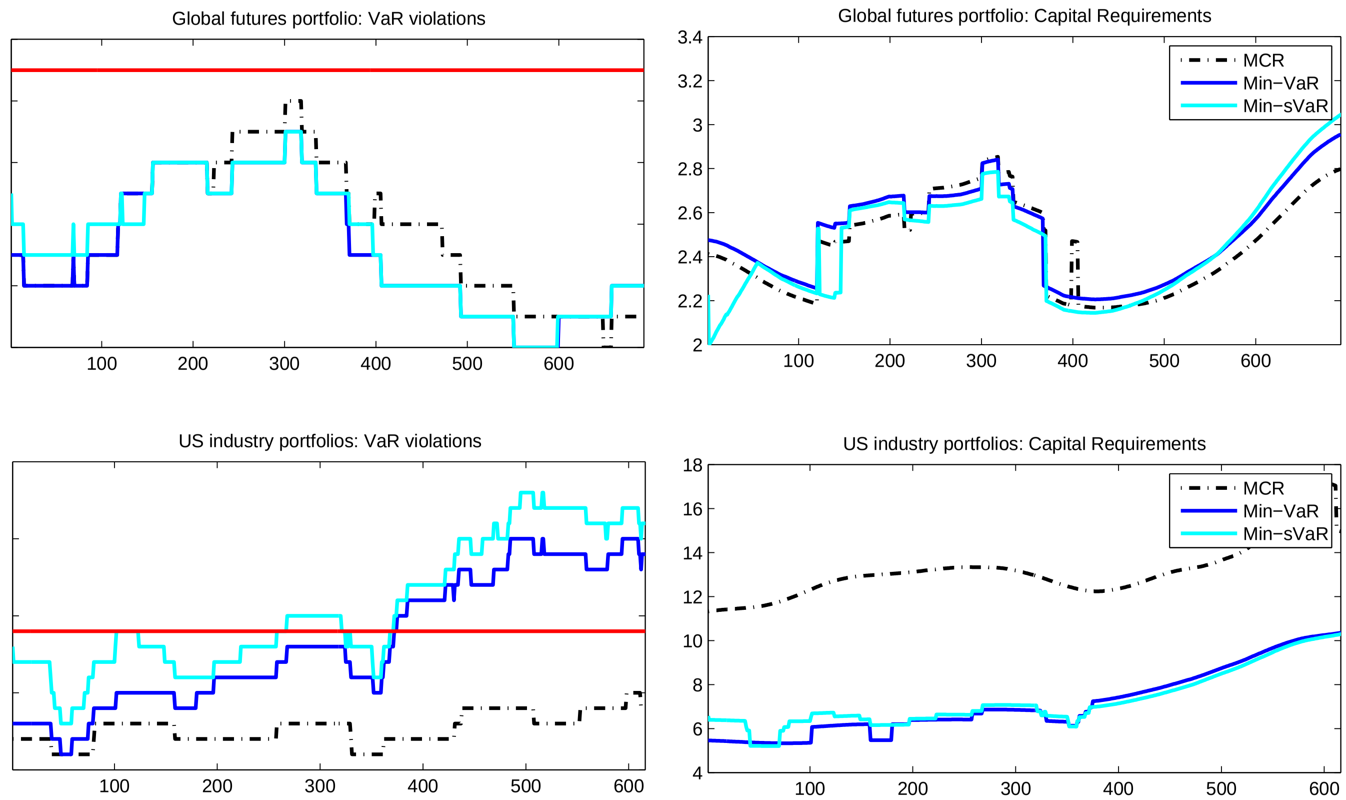

Figure 2: Number of VaR violations (left) and capital requirements (right) for the MCR (dashed line), Min-VaR (solid blue line), and Min-sVaR (solid cyan line) portfolios under stress scenario 2 when the DCC specification is used to model the conditional covariance matrix. The horizontal line indicates the “red zone” threshold (9 violations in the previous 250 trading days).

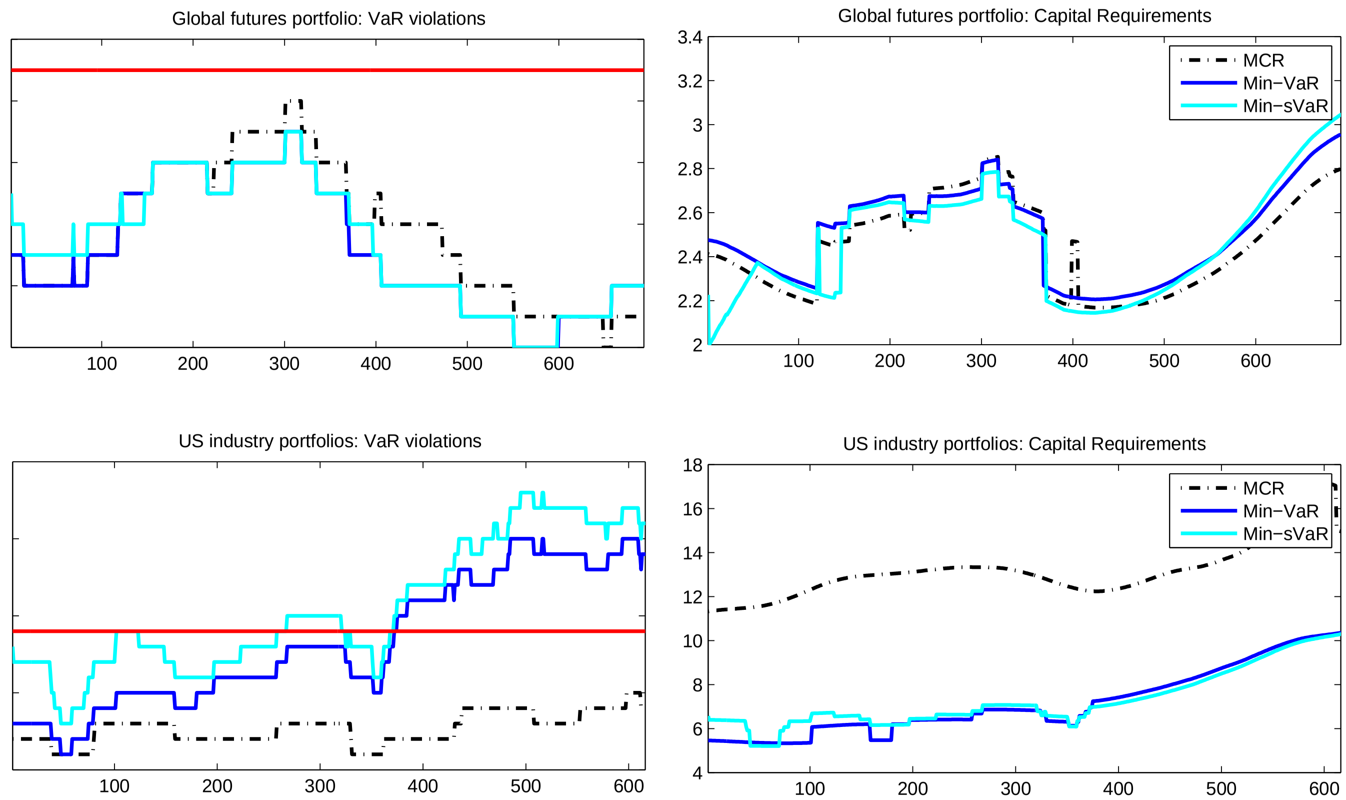

Figure 3: Number of VaR violations (left) and capital requirements (right) for the MCR (dashed line), Min-VaR (solid blue line), and Min-sVaR (solid cyan line) portfolios under stress scenario 3 when the DCC specification is used to model the conditional covariance matrix. The horizontal line indicates the “red zone” threshold (9 violations in the previous 250 trading days).